## MATHEMATICS



## Y6 Multiplication and Division

6327
Square numbers, prime numbers and identifying factors

## Equipment

Paper, pencil, ruler.

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## Concepts

The term 'square number' needs to be recognised and understood.
$4 \times 4$ can be written as $4^{2} \quad$ This is pronounced, " four squared".
$\mathbf{7 \times 7}$ can be written as $\mathbf{7}^{\mathbf{2}}$ This is pronounced, " seven squared".
Many children are confused with this and believe that the squared sign ${ }^{2}$ means multiply by two - which is incorrect.

If the square of $\mathbf{4}$ or four squared is 16 , then the square root of 16 is four. Square roots is an interesting topic which children enjoy and makes sense to introduce it at the same time as square numbers.

If $\boldsymbol{7}^{\mathbf{2}}=\mathbf{7 \times 7 = 4 9}$, then the square root of $\mathbf{4 9}$ is $\mathbf{7}$.
The square root sign is $\sqrt{ } \quad$ so $\sqrt{49}$ is 7
Multiples of numbers are numbers which are produced by multiplying that number by another whole number. E.g. 4, 6, 8 are multiples of 2.

The factors of a number (e.g. 24) are those numbers which divide exactly into it.
The factors of 24 are $1,2,3,4,6,8,12,24$.
Every whole number has at least two factors - one and the number itself.

A number which only has two factors (one and itself) is called a prime number.
The first prime numbers are 2, 3, 5, 7, 11, 13, 17 and 19.
( Note that 1 is not usually considered as a prime number.)
To decide whether a number is prime a check has to be made as to whether it has any factors.

Try dividing by the first few prime numbers, 2, 3, 5, 7, 11. See examples on sheets.
( See also sheets on multiples and tests of divisibility.)

## SQUARE NUMBERS

You should already know the square numbers up to 100.
Reminder: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
Below is a tables chart from $10 \times 10$ up to $20 \times 20$.
(larger version available on page 15.)
You may not have seen one of these before, but it works just like the normal tables chart from 1 to 10.

Begin by shading the answer to $10 \times 10$.
Now shade $11 \times 11$ and then $12 \times 12$.
Continue up to $20 \times 20$. The first two have been shaded for you.

|  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| 11 | 110 | 121 | 132 | 143 | 154 | 165 | 176 | 187 | 198 | 209 | 220 |
| 12 | 120 | 132 | 144 | 156 | 168 | 180 | 192 | 204 | 216 | 228 | 240 |
| 13 | 130 | 143 | 156 | 169 | 182 | 195 | 208 | 221 | 234 | 247 | 260 |
| 14 | 140 | 154 | 168 | 182 | 196 | 210 | 224 | 238 | 252 | 266 | 280 |
| 15 | 150 | 165 | 180 | 195 | 210 | 225 | 240 | 255 | 270 | 285 | 300 |
| 16 | 160 | 176 | 192 | 208 | 224 | 240 | 256 | 272 | 288 | 304 | 320 |
| 17 | 170 | 187 | 204 | 221 | 238 | 255 | 272 | 289 | 306 | 323 | 340 |
| 18 | 180 | 198 | 216 | 234 | 252 | 270 | 288 | 306 | 324 | 342 | 360 |
| 19 | 190 | 209 | 228 | 247 | 266 | 285 | 304 | 323 | 342 | 361 | 380 |
| 20 | 200 | 220 | 240 | 260 | 280 | 300 | 320 | 340 | 360 | 380 | 400 |

1. Write down all the numbers that you have shaded.
2. What do you notice about the pattern that you have shaded?

These numbers are all square numbers.
They are made by multiplying a whole number by itself.
Write down the answers to these sums:
3. $13 \times 13=$
4. $15 \times 15=$
5. $17 \times 17=$
6. $19 \times 19=$
7. $12 \times 12=$
8. $14 \times 14=$
9. $16 \times 16=$
10. $18 \times 18=$

All the answers above are square numbers.

## SQUARE NUMBERS

The square of 10 is 100

$$
10^{2}=100
$$

The square of 11 is 121

$$
11^{2}=121
$$

In the same way, fill in the gaps in these statements:

1. The square of 15 is $\square$

$$
15^{2}=225
$$

2. The square of $\square$ is 100
3. The square of 13 is $\square$

$$
10^{2}=100
$$

$$
13^{2}=\square
$$

4. The square of 8 is $\square$
5. The square of 20 is $\square$

$$
\square=64
$$

$20^{2}=$ $\square$

## Square roots

If the square of $\mathbf{5}$ is $\mathbf{2 5}$ then the square root of $\mathbf{2 5}$ is $\mathbf{5}$.
The sign for square root is $\sqrt{ }$ so $\sqrt{49}$ is 7
If the square of $\mathbf{4}$ is $\mathbf{1 6}$ then the square root of $\mathbf{1 6}$ is $\mathbf{4}$ so $\sqrt{16}=\mathbf{4}$
Try to answer these questions:
6. What is the square root of 36 ?
7. $\sqrt{81}=\square$
8. What is the square root of 121 ?
9. $\sqrt{225}=\square$
10. The square root of 400 is $\square$


## SQUARE NUMBERS

Try to learn the set of square numbers up to 20 squared.
Start with the first 10 square numbers. They are:
$1, \quad 4, \quad 9, \quad 16, \quad 25, \quad 36,49,64, \quad 81,100$

Then try and learn the next five square numbers. They are:
121, 144, 169, 196, 225
This pattern or sequence of numbers keeps coming up in maths. Make sure that you can recognise it as square numbers.

You should also be able to say what the square roots of these numbers are. However, with larger numbers it is more difficult to recognise square roots, but it is quite simple to use a calculator.

Most calculators have a square root button. $(\sqrt{ })$
To find the square root of a number, just type in the number and then press the square root sign. You do not even have to press the equals sign.

Using a calculator find the square roots of these numbers:
(Before you use the calculator, make a prediction!)

|  | Number | Predicted square <br> root | Square root |
| :--- | :---: | :---: | :---: |
| 1. | 400 |  |  |
| 2. | 900 |  |  |
| 3. | 3600 |  |  |
| 4. | 4900 |  |  |
| 5. | 12100 |  |  |
| 6. | 10000 |  |  |
| 7. | 361 |  |  |
| 8. | 625 |  |  |

## SQUARE NUMBERS

List the first ten ODD NUMBERS as shown below:

$$
1,3,5,7,9,11,13,15,17,19 .
$$

Add the first two odd numbers together: $1+3=4$
Write out a chart like the one below and fill in the answers as shown:

| $1+3=$ | 4 | $2 \times 2=4$ | $2^{2}=4$ |
| :--- | :--- | :--- | :--- |
| $1+3+5=$ | 9 | $3 \times 3=9$ | $3^{2}=9$ |
| $1+3+5+7=$ |  |  | $4^{2}$ |
| $1+$ |  |  | $5^{2}$ |
| $1+$ |  |  | $6^{2}$ |
| $1+$ |  |  | $7^{2}$ |
| $1+$ |  |  | $8^{2}$ |

1. What patterns do you notice? Describe them in words.
2. Which of the numbers below are square numbers?
24
2
400
25
50
81
3. Which of the numbers below has a whole number square root? (ie the square root is a whole number, with no decimal or remainder)
6
9
8
12
25
88
4. What is the square root of 144 ?
5. If the square of 4 is added to the square root of 25 what is the answer?
6. What is the square root of 169 ?
7. What is three squared plus four squared? (Clue: it is not seven squared!)
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## MULTIPLES

Remember: you can get a MULTIPLE of a whole number by multiplying that number by another whole number.

The answers to all your 'times tables' are multiples.
e.g. $8,12,16,20$, etc are all MULTIPLES of 4.

12, 18, 24, 30, etc are all MULTIPLES of 6 .

1. Write down the first ten multiples of 11.
2. What do you notice?
3. Which of these numbers are multiples of 7 ?

$$
71, \quad 47, \quad 76, \quad 49, \quad 56
$$

4. Write down the first ten multiples of 20 .
5. What do you notice? Look especially at the tens digit.
6. Which of these numbers are multiples of 9 ?

36, 47, 91, 99, 92
7. Write down the first ten multiples of 12.
8. What do you notice? Is there a pattern?
9. Write down which of these numbers are multiples of 6 :
25,
46,
56, 66,
76,
86,
90
10. Which of these numbers are multiples of both 3 and 5 ?
45,
50,
25,
15,
60,
10,
11

## FACTORS

Reminder: the factors of a number are those numbers that divide exactly into it, without leaving a remainder.

> e.g. the factors of 20 are: $1,2, \quad 4, \quad 5, \quad 10 \quad$ and 20 , because: $1 \times 20=20$ and $2 \times 10=20 \quad$ and $4 \times 5=20$

Every number has a factor of 1 , because 1 will divide exactly into any whole number.
Every number has itself as a factor, because any whole number will divide into itself exactly once.

## Reminder: how do you find the factors of a number?

Let's try 50:
a. First of all write down 1 and the number itself as factors.
$(1,50)$
b. Then see if 2 will divide exactly into the number. The answer is yes,
$2 \times 25=50$ so 2 and 25 are factors.
(1, 2, 25, 50)
c. Then try 3. No, 3 does not go into 50 exactly.
d. Carry on with each number, 4 next.

No, 4 will not divide exactly into 50 .
e. 5 will go into 50 exactly, because $5 \times 10=50$.

So 5 and 10 are both factors of 50 .
( $1,2,5,10,25,50$ )
f. 6,7 and 8 will not divide exactly into 50 .
g. We have reached as far as we need to go, because $8 \times 8=64$ which is more than 50. This means we have covered all possibilities.

The factors of 50 are $1,2,5,10,25$ and 50
In the same way, find the factors of:

1. 49
2. 100
3. 67
4. 44
5. 99

## FACTORS

Remember, the factors of a number are the numbers that will divide exactly into it, without a remainder.

If you know your tables you should find this work easy.
Find all the factors of these numbers:

1. 35
2. 21
3. 57
4. 80
5. 27

> Let's see, start with one....then two....then three....
6. 48
7. 31
8. 26
9. 71

10. 72

Fill in the missing factors:
11. The factors of 18 are: 1 , $\square$ 3, 6 , $\square$
12. The factors of 36 are:

1, 2, 3,

$9, \quad 12$, $\square$ 36
13. The factors of 19 are: $\square$
$\square$
14. The factors of 55 are: 1,5 , $\square$ 55
15. The factors of 81 are: $\square$ 3, 9 , $\square$ , $\square$

## PRIME NUMBERS

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

1. Except for the number 2, cross out all the numbers which divide by two. That is, all the even numbers.
2. Except for the number 3, cross out all the numbers which divide by three. (Think of your 3 times table! Some have been crossed out already!)
3. Except for the number 5 , cross out all the numbers which divide by five. ( Think of your 5 times table! )
4. The next number which is not crossed off is 7 . Leave that uncrossed, but go through the square crossing off any numbers which are a multiple of 7 . ( Some are crossed off already!)
5. List all the numbers which have not been crossed off. They are all the PRIME numbers up to 100 . Check you have them correctly written down.
6. Try to learn this list of numbers. It will prove to be very useful later on.

## PRIME NUMBERS

A number which only has factors of 1 and itself is called a PRIME NUMBER.
For example the factors of 7 are 1 and 7 . There is no other way of multiplying two whole numbers to make 7 .
So 7 is a prime number.

## How to find whether a larger number is prime.

There is a quick way to check whether larger numbers are prime without dividing all lower numbers into it.

Start by seeing if the first few prime numbers will divide exactly.
These are: 2, 3, 5, 7, and 11.
There is no need to divide by any of the other numbers between one and eleven. Example:

Is 39 a prime number?
Does 39 divide exactly by 2 ? No, because it is an odd number.
Does 39 divide exactly by 3 ? Yes, $3 \times 13=39$.
39 is not a prime number.
Is 53 a prime number?
Does 53 divide exactly by 2 ? No, because it is an odd number.
Does 53 divide exactly by 3 ? No.
Does 53 divide exactly by 5 ? No, because it does not end in 0 or 5 .
Does 53 divide exactly by 7 ? No.
There is no need to go any further because $8 \times 8$ is 64 , which is more than 53 .
53 is a prime number.

1. In the same way as above check the numbers between 50 and 100 to see which are prime. Write them down.

## Problems

1. Mrs Green has 30 children in her class. She is very keen that she has groups of equal sizes, but she does not mind how big or small the groups are. List all the possible group sizes that she could have.
2. Mr. Hardman is the PE teacher. He hates taking class 5 T because there are 29 children in this class. Explain why he might not like taking this class for a football lesson.
3. A PERFECT NUMBER is a number where all the factors of the number (apart from itself!) add up to the same total as the number itself.

6 is a perfect number because the factors of 6 are $1,2,3$ and 6 $1+2+3=6$

There is only one other perfect number between 10 and 30. Can you find it?
4. Mr and Mrs Posh are holding a magnificent wedding reception for their daughter, Pru. They have invited 40 guests. All the tables must be the same size and have the same number of guests on them. List the possible combinations that they could have for their guests. For instance they could have one huge table with all 40 guests on it.
5. Which numbers in the box below are prime?

| 12 | 21 | 31 | 55 | 61 | 81 |
| :--- | :--- | :--- | :--- | :--- | :--- |

6. The number 496 is PERFECT. Find all the factors of 496 and see if this statement is true. Use a calculator to help you.
7. A piece of paper has an area of 24 sq cm . If the sides are an exact number of cm ., what possible lengths could the paper be?

## Answers

## Page 3

1. $144,169,196,225,256,289,324,361,400 \quad$ 2. Diagonal, left to right.
2. 169
3. 225
4. 289
5. 361
6. 144
7. 196
8. 256
9. 324

Page 4

1. 225
2. 10
3. 169,169
4. $64,8^{2}$
5. 400,400
6. 6
7.9
7. 11
8. 15
9. 20

Page 5

1. 20
2. 30
3. 60
4. 70
5. 110
6. 100
7. 19
8. 25

Page 6

| $1+3=$ | 4 | $2 \times 2=4$ | $2^{2}=4$ |
| :--- | :--- | :--- | :--- |
| $1+3+5=$ | 9 | $3 \times 3=9$ | $3^{2}=9$ |
| $1+3+5+7=$ | 16 | $4 \times 4=16$ | $4^{2}=16$ |
| $1+3+5+7+9=$ | 25 | $5 \times 5=25$ | $5^{2}=25$ |
| $1+3+5+7+9+11=$ | 36 | $6 \times 6=36$ | $6^{2}=36$ |
| $1+3+5+7+9+11+13=$ | 49 | $7 \times 7=49$ | $7^{2}=49$ |
| $1+3+5+7+9+11+13+15=$ | 64 | $8 \times 8=64$ | $8^{2}=64$ |

1. adding next odd number, square number patterns $4,9,16,25,36,49,64$
2. $400,25,81$
3. 9,25
4. 12
5. 21
6. 13
7.25

## Page 7

1. $11,22,33,44,55,66,77,88,99,110$ 2. Going up in ones until 9 . Repeating digits. 3. 49,56 4. $20,40,60,80,100,120,140,160,180,200$ 5. Tens digit going up in twos. 6.36 99 7.12, 24, 36, 48, 60, 72, 84, 96, 108, $120 \quad$ 8. All even numbers in units 9.66, $90 \mathbf{1 0 . 4 5 , 1 5 , 6 0}$

Page 8

1. 1, 7, 49 2. $1,2,4,5,10,20,25,50,100$ 3. $1,67 \quad$ 4. $1,2,4,11,22,44$
2. $1,3,9,11,33,99$

## Page 9

1. 1, 5, 7, 35
2. $1,3,7,21$
3. 1, 3, 19, 57
4. $1,2,4,5,8,10,16,20,40,80$
5. 1, 3, 9, 27
6. $1,2,3,4,6,8,12,16,24,48$
7. 1, 31
8. $1,2,13,26$
9. $1,71 \quad$ 10. $1,2,3,4,6,8,9,12,18,24,36,72$
11.2, 18
10. 6,18
11. 1,19
12. 11
13. 1, 27, 81

## Page 10

1. (1) $2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61$, $67,71,73,79,83,89,97$

## Page 11.

1. $59,61,67,71,73,79,83,89,97$

Page 12

1. $2,3,5,6,10,15,30$ ( is one a group?) 2.29 is a prime number so Mr Hardman can not make even sided teams. 3. 28 4. 1, 2, 4, 5, 8, 10, 20, 40 5.31, 61
2. True $1,2,4,8,16,31,62,124,248$, adds up to 496
3. $1 \mathrm{~cm}, 2 \mathrm{~cm}, 3 \mathrm{~cm}, 4 \mathrm{~cm}, 6 \mathrm{~cm}, 8 \mathrm{~cm}, 12 \mathrm{~cm}, 24 \mathrm{~cm}$

|  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| 11 | 110 | 121 | 132 | 143 | 154 | 165 | 176 | 187 | 198 | 209 | 220 |
| 12 | 120 | 132 | 144 | 156 | 168 | 180 | 192 | 204 | 216 | 228 | 240 |
| 13 | 130 | 143 | 156 | 169 | 182 | 195 | 208 | 221 | 234 | 247 | 260 |
| 14 | 140 | 154 | 168 | 182 | 196 | 210 | 224 | 238 | 252 | 266 | 280 |
| 15 | 150 | 165 | 180 | 195 | 210 | 225 | 240 | 255 | 270 | 285 | 300 |
| 16 | 160 | 176 | 192 | 208 | 224 | 240 | 256 | 272 | 288 | 304 | 320 |
| 17 | 170 | 187 | 204 | 221 | 238 | 255 | 272 | 289 | 306 | 323 | 340 |
| 18 | 180 | 198 | 216 | 234 | 252 | 270 | 288 | 306 | 324 | 342 | 360 |
| 19 | 190 | 209 | 228 | 247 | 266 | 285 | 304 | 323 | 342 | 361 | 380 |
| 20 | 200 | 220 | 240 | 260 | 280 | 300 | 320 | 340 | 360 | 380 | 400 |


|  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |
| 11 | 110 | 121 | 132 | 143 | 154 | 165 | 176 | 187 | 198 | 209 | 220 |
| 12 | 120 | 132 | 144 | 156 | 168 | 180 | 192 | 204 | 216 | 228 | 240 |
| 13 | 130 | 143 | 156 | 169 | 182 | 195 | 208 | 221 | 234 | 247 | 260 |
| 14 | 140 | 154 | 168 | 182 | 196 | 210 | 224 | 238 | 252 | 266 | 280 |
| 15 | 150 | 165 | 180 | 195 | 210 | 225 | 240 | 255 | 270 | 285 | 300 |
| 16 | 160 | 176 | 192 | 208 | 224 | 240 | 256 | 272 | 288 | 304 | 320 |
| 17 | 170 | 187 | 204 | 221 | 238 | 255 | 272 | 289 | 306 | 323 | 340 |
| 18 | 180 | 198 | 216 | 234 | 252 | 270 | 288 | 306 | 324 | 342 | 360 |
| 19 | 190 | 209 | 228 | 247 | 266 | 285 | 304 | 323 | 342 | 361 | 380 |
| 20 | 200 | 220 | 240 | 260 | 280 | 300 | 320 | 340 | 360 | 380 | 400 |

